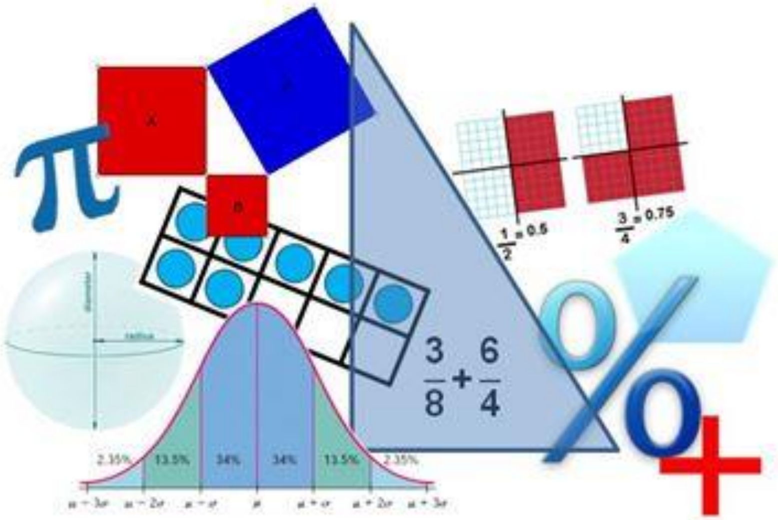
Mathematics

2016 Standards of Learning

Grade 5

Curriculum Framework



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Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills
This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient, and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand, and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> "Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Mathematics instruction in grades three through five should continue to foster the development of number sense, with greater emphasis on decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction in grades three through five allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names), and making connections among mathematical concepts as well as to other content areas. Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve a variety of problem types.

5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

Understanding the Standard	Essential Knowledge and Skills
• The structure of the base-ten number system is based upon a simple pattern of tens in which each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship. To investigate this relationship, use base-ten proportional manipulatives, such as place value mats/charts, decimal squares, base-ten blocks, meter sticks, as well as the ten-to-one non-proportional model, and money. • A decimal point separates the whole number places from the places less than one. Place values extend infinitely in two directions from a decimal point. A number containing a decimal point is called a *decimal number* or simply a *decimal*. • To read decimals, – read the whole number to the left of the decimal point; – read the decimal point as "and"; – read the digits to the right of the decimal point just as you would read a whole number; and – say the name of the place value of the digit in the smallest place. • Any decimal less than one will include a leading zero (e.g., 0.125). This number may be read as "zero and one hundred twenty-five thousandths" or as "one hundred twenty-five thousandths." • Decimals can be rounded in situations when exact numbers are not needed. Strategies for rounding whole numbers can be applied to rounding decimals. • Number lines are tools that can be used in developing a conceptual understanding of rounding decimals. One strategy includes creating a number line that shows the decimal that is to be rounded. Locate it on the number line. Next, determine the closest multiples of whole numbers, tenths, or hundredth, it is between. Then, identify to which it is closer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a decimal through thousandths, round to the nearest whole number, tenth, or hundredth.

5.2 The student will

a) represent and identify equivalencies among fractions and decimals, with and without models; * and

b) compare and order fractions, mixed numbers, and/or decimals, in a given set, from least to greatest and greatest to least.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Students should focus on determining equivalent decimals of familiar fractions with denominators that are factors of 100 making connections to tenths and hundredths. (e.g., $\frac{2}{5}$ = $\frac{4}{10}$ or 0.4) and (e.g., $\frac{7}{20}$ = $\frac{35}{100}$ or 0.35). • Students should have experience with fractions such as $\frac{1}{8}$, whose decimal representation is a terminating decimal (e. g., $\frac{1}{8}$ = 0.125) and with fractions such as $\frac{2}{3}$, whose decimal representation does not end but continues to repeat (e. g., $\frac{2}{3}$ = 0.666…). The repeating decimal can be written with an ellipsis (three dots) as in 0.666… or denoted with a bar above the digits that repeat as in $0.\overline{6}$. • To help students compare the value of two decimals through thousandths, use manipulatives, such as place value mats/charts, 10-by-10 grids, decimal squares, base-ten blocks, meter sticks, number lines, and money. • Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). • An amount less than one whole can be represented by a fraction or by an equivalent decimal. • Base-ten models (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, money) demonstrate the relationship between fractions and decimals.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form with concrete or pictorial models. (a) • Represent decimals in their equivalent fraction form (thirds, eighths, and factors of 100) with concrete or pictorial models. (a) • Identify equivalent relationships between decimals and fractions with denominators that are thirds, eighths, and factors of 100 in their equivalent decimal form without models. (a) • Compare and order from least to greatest and greatest to least a given set of no more than four decimals, fractions (proper or improper), and/or mixed numbers with denominators of 12 or less. (b) • Use the symbols >, <, =, and ≠ to compare decimals through thousandths, fractions (proper or improper fractions), and/or mixed numbers, having denominators of 12 or less. (b)

5.3 The student will
a) identify and describe the characteristics of prime and composite numbers; and
b) identify and describe the characteristics of even and odd numbers.

Understanding the Standard	Essential Knowledge and Skills
• Natural numbers are the counting numbers starting at one. • A prime number is a natural number, other than one, that has exactly two different factors, one and the number itself. • A composite number is a natural number that has factors other than one and itself. • The number one is neither prime nor composite because it has only one set of factors and both factors are one. • The prime factorization of a number is a representation of the number as the product of its prime factors. For example, the prime factorization of 18 is 2 × 3 × 3. • Prime factorization concepts can be developed by using factor trees. • Prime or composite numbers can be represented by rectangular models or rectangular arrays on grid paper. A prime number can be represented by only one rectangular array (e.g., seven can be represented by a 7 × 1 and a 1 × 7). A composite number can always be represented by two or more rectangular arrays (e.g., nine can be represented by a 9 × 1, a 1 × 9, or a 3 × 3). • Divisibility rules are useful tools in identifying prime and composite numbers. • Odd and even numbers can be explored in different ways (e.g., dividing collections of objects into two equal groups or pairing objects). When pairing objects, the number of objects is even when each object has a pair or partner. When an object is left over, or does not have a pair, then the number is odd. • Students should use manipulatives (e.g., base-ten blocks, cubes, tiles, hundreds board, etc.) to explore and categorize numbers into groups of odd or even. • Examples of ways to use manipulatives to show even and odd numbers may include (but are not limited to): – for an even number, such as 12, six pairs of counters can be formed with no remainder, or two groups of six counters can be formed with no remainder; and – for an odd number, such as 13: (a) six pairs of counters can be formed with one counter remaining, or (b) two groups of six counters can be formed with one counter remaining.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify prime numbers less than or equal to 100. (a) • Identify composite numbers less than or equal to 100. (a) • Demonstrate with concrete or pictorial representations and explain orally or in writing why a number is prime or composite. (a) • Identify which numbers are even or odd. (b) • Demonstrate with concrete or pictorial representations and explain orally or in writing why a number is even or odd. (b) • Demonstrate with concrete or pictorial representations and explain orally or in writing why the sum or difference of two numbers is even or odd. (b)

5.3 The student will

a) identify and describe the characteristics of prime and composite numbers; and

b) identify and describe the characteristics of even and odd numbers.

Understanding the Standard	Essential Knowledge and Skills
• Students should use rules to categorize numbers into groups of odd or even. Rules can include: – An odd number does not have two as a factor and is not divisible by two. – The sum of two even numbers is even. – The sum of two odd numbers is even. – The sum of an even number and an odd number is odd. – Even numbers have an even number or zero in the ones place. – Odd numbers have an odd number in the ones place. – An even number has two as a factor and is divisible by two. – The product of two even numbers is even. – The product of two odd numbers is odd. – The product of an even number and an odd number is even.	

Computation and estimation in grades three through five should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades. Multiplication and division with decimals will be explored in grade five.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability to identify and use relationships among operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use properties of operations to solve problems (e.g., 7 × 28 is equivalent to (7 × 20) + (7 × 8)).

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than one because both fractions are less than $\frac{1}{2}$). Using estimation, students should develop strategies to recognize the reasonableness of their solutions.

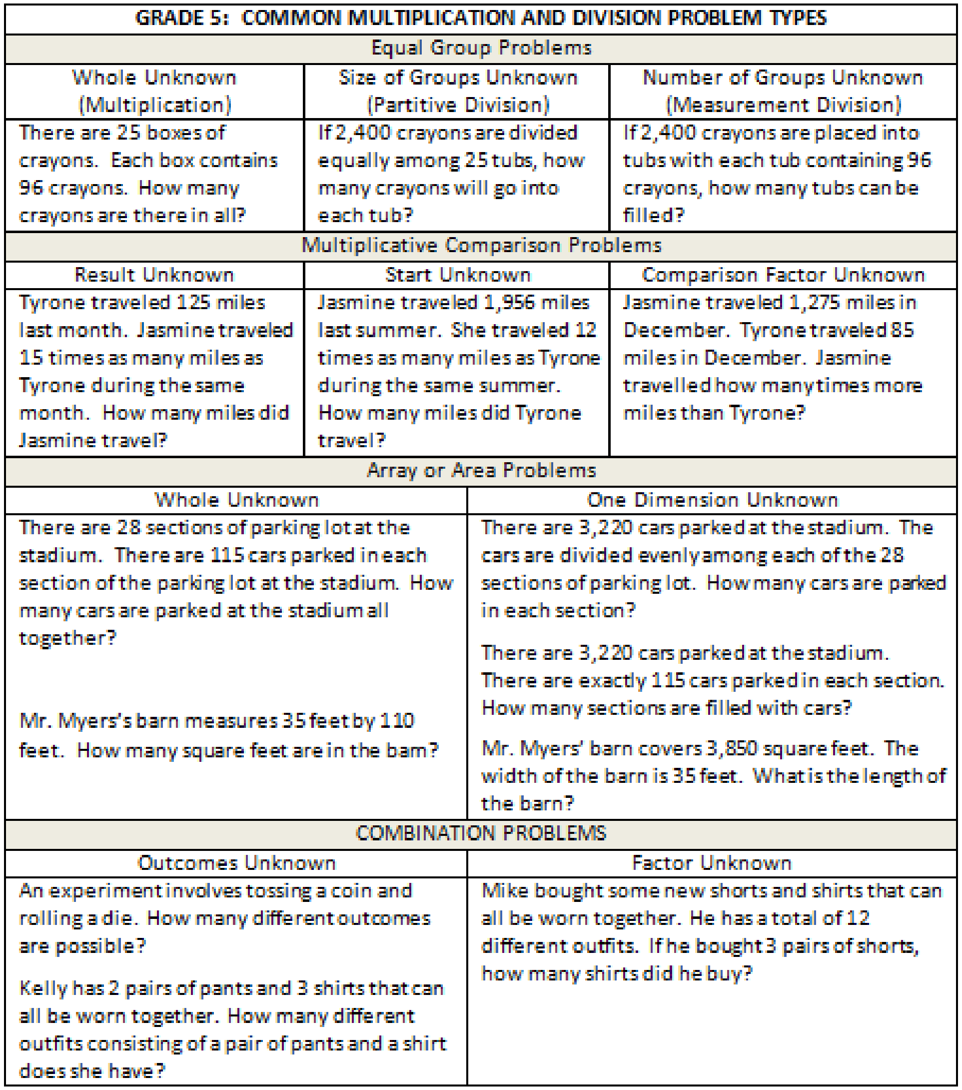
Additionally, students should enhance their ability to select an appropriate problem-solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers.

Understanding the Standard	Essential Knowledge and Skills
• The problem-solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all, altogether, difference,* etc., encourages students to perform a particular operation rather than make sense of the context of the problem. A key-word focus prepares students to solve a limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses. • Estimation can be used to determine a reasonable range for the answer to computation and to verify the reasonableness of sums, differences, products, and quotients of whole numbers. • The least number of steps necessary to solve a single-step problem is one. • A multistep problem incorporates two or more operational steps (operations can be the same or different). • Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types. • Students should experience a variety of problem types related to multiplication and division. Some examples are included in the following chart:	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Create single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers, with and without remainders. • Estimate the sum, difference, product, and quotient of whole numbers. • Apply strategies, including place value and application of the properties of addition and multiplication, to solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers, with and without remainders, in which: – sums, differences, and products do not exceed five digits; – factors do not exceed two digits by three digits; – divisors do not exceed two digits; or – dividends do not exceed four digits. • Use the context of a practical problem to interpret the quotient and remainder.

5.4 **The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers.**

Understanding the Standard	Essential Knowledge and Skills



<table>
<tr><th colspan="6">GRADE 5: COMMON MULTIPLICATION AND DIVISION PROBLEM TYPES</th></tr>
<tr><th colspan="6">Equal Group Problems</th></tr>
<tr><th colspan="2">Whole Unknown (Multiplication)</th><th colspan="2">Size of Groups Unknown (Partitive Division)</th><th colspan="2">Number of Groups Unknown (Measurement Division)</th></tr>
<tr><td colspan="2">There are 25 boxes of crayons. Each box contains 96 crayons. How many crayons are there in all?</td><td colspan="2">If 2,400 crayons are divided equally among 25 tubs, how many crayons will go into each tub?</td><td colspan="2">If 2,400 crayons are placed into tubs with each tub containing 96 crayons, how many tubs can be filled?</td></tr>
<tr><th colspan="6">Multiplicative Comparison Problems</th></tr>
<tr><th colspan="2">Result Unknown</th><th colspan="2">Start Unknown</th><th colspan="2">Comparison Factor Unknown</th></tr>
<tr><td colspan="2">Tyrone traveled 125 miles last month. Jasmine traveled 15 times as many miles as Tyrone during the same month. How many miles did Jasmine travel?</td><td colspan="2">Jasmine traveled 1,956 miles last summer. She traveled 12 times as many miles as Tyrone during the same summer. How many miles did Tyrone travel?</td><td colspan="2">Jasmine traveled 1,275 miles in December. Tyrone traveled 85 miles in December. Jasmine travelled how many times more miles than Tyrone?</td></tr>
<tr><th colspan="6">Array or Area Problems</th></tr>
<tr><th colspan="3">Whole Unknown</th><th colspan="3">One Dimension Unknown</th></tr>
<tr><td colspan="3">There are 28 sections of parking lot at the stadium. There are 115 cars parked in each section of the parking lot at the stadium. How many cars are parked at the stadium all together?
Mr. Myers's barn measures 35 feet by 110 feet. How many square feet are in the barn?</td><td colspan="3">There are 3,220 cars parked at the stadium. The cars are divided evenly among each of the 28 sections of parking lot. How many cars are parked in each section?
There are 3,220 cars parked at the stadium. There are exactly 115 cars parked in each section. How many sections are filled with cars?
Mr. Myers' barn covers 3,850 square feet. The width of the barn is 35 feet. What is the length of the barn?</td></tr>
<tr><th colspan="6">COMBINATION PROBLEMS</th></tr>
<tr><th colspan="3">Outcomes Unknown</th><th colspan="3">Factor Unknown</th></tr>
<tr><td colspan="3">An experiment involves tossing a coin and rolling a die. How many different outcomes are possible?
Kelly has 2 pairs of pants and 3 shirts that can all be worn together. How many different outfits consisting of a pair of pants and a shirt does she have?</td><td colspan="3">Mike bought some new shorts and shirts that can all be worn together. He has a total of 12 different outfits. If he bought 3 pairs of shorts, how many shirts did he buy?</td></tr>
</table>

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers.

Understanding the Standard	Essential Knowledge and Skills
- Students also need exposure to various types of practical problems in which they must interpret	

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers.

Understanding the Standard

the quotient and remainder based on the context. The chart below includes one example of each type of problem.

MAKING SENSE OF THE REMAINDER IN DIVISION

TYPE OF PROBLEM	EXAMPLE
Remainder is not needed and can be left over (or discarded).	Bill has 29 pencils to share fairly with 6 friends. How many pencils will each friend receive? 4 pencils with 5 pencils left over
Remainder is partitioned and represented as a fraction or decimal.	Six friends will share 29 ounces of juice. How many ounces will each person get if all of the juice is shared equally? $4\frac{5}{6}$ ounces
Remainder forces the answer to be increased to the next whole number.	There are 29 people going to the party by car. How many cars will be needed if each car holds 6 people? 5 cars
Remainder forces the answer to be rounded (giving an approximate answer).	Six children will share a bag of candy containing 29 pieces. About how many pieces of candy will each child get? about 5 pieces of candy

- Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers.
- Grade five students should explore and apply the properties of addition and multiplication as strategies for solving addition, subtraction, multiplication, and division problems using a variety of representations (e.g., manipulatives, diagrams, and symbols).
- The properties of the operations are “rules” about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties are most appropriate for exploration at this level:
  - The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). Similarly, the commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 2 × 3 = 3 × 2).
  - The identity property of addition states that if zero is added to a given number, the sum is the

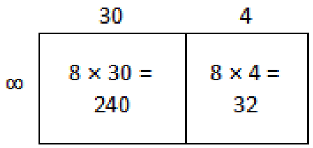
Essential Knowledge and Skills

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers.

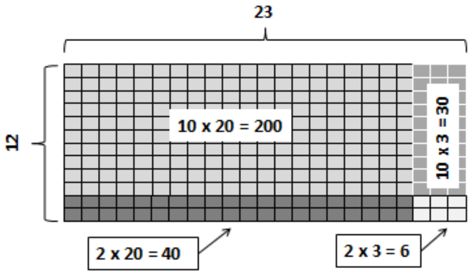
Understanding the Standard

same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number.

- The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., $15 + (35 + 16) = (15 + 35) + 16$).
- The associative property of multiplication states that the product stays the same when the grouping of factors is changed (e.g., $6 \times (3 \times 5) = (6 \times 3) \times 5$).
- The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products.
- Examples of the distributive property include:
  - $3(9) = 3(5 + 4)$
  - $3(54 + 4) = 3 \times 54 + 3 \times 4$
  - $5 \times (3 + 7) = (5 \times 3) + (5 \times 7)$
  - $(2 \times 3) + (2 \times 5) = 2 \times (3 + 5)$
  - 9×23
    $9(20 + 3)$
    $180 + 27$
    207
  - 34×8



  - 23×12
    $(20 + 3) \times (10 + 2)$
    $(20 \times 10) + (20 \times 2) + (3 \times 10) + (3 \times 2)$
    $200 + 40 + 30 + 6$
    276



Essential Knowledge and Skills

5.5 **The student will**

a) estimate and determine the product and quotient of two numbers involving decimals* and

b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and create and solve single-step practical problems involving division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard

- Addition and subtraction of decimals may be investigated using a variety of models (e.g., 10-by-10 grids, number lines, money).
- The base-ten relationships and procedures developed for whole number computation apply to decimal computation, giving careful attention to the placement of the decimal point in the solution.
- In cases where an exact product is not required, the product of decimals can be estimated using strategies for multiplying whole numbers, such as front-end and compatible numbers, or rounding. In each case, the student needs to determine where to place the decimal point to ensure that the product is reasonable.
- Estimation keeps the focus on the meaning of the numbers and operations, encourages reflective thinking, and helps build informal number sense with decimals. Students can reason with benchmarks to get an estimate without using an algorithm.
- Estimation can be used to determine a reasonable range for the answer to computation and to verify the reasonableness of sums, differences, products, and quotients of decimals.
- Division is the operation of making equal groups or shares. When the original amount and the number of shares are known, divide to determine the size of each share. When the original amount and the size of each share are known, divide to determine the number of shares. Both situations may be modeled with base-ten manipulatives.
- The fair-share concept of decimal division can be modeled, using manipulatives (e.g., base-ten blocks). Multiplication and division of decimals can be represented with arrays, paper folding, repeated addition, repeated subtraction, base-ten models, and area models.
- Students in grade four studied decimals through thousandths and solved practical problems that involved addition and subtraction of decimals. Consideration should be given to creating division problems with decimals that do not exceed quotients in the thousandths. Teachers may desire to work backwards in creating appropriate decimal division problems meeting the parameters for grade five students.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Estimate and determine the product of two numbers in which:
  - the factors do not exceed two digits by two digits (e.g., 2.3 × 4.5, 0.08 × 0.9, 0.85 × 2.3, 1.8 × 5); and
  - the products do not exceed the thousandths place. (Leading zeroes will not be considered when counting digits.) (a)
- Estimate and determine the quotient of two numbers in which
  - quotients do not exceed four digits with or without a decimal point;
  - quotients may include whole numbers, tenths, hundredths, or thousandths;
  - divisors are limited to a single digit whole number or a decimal expressed as tenths; and
  - no more than one additional zero will need to be annexed. (a)
- Use multiple representations to model multiplication and division of decimals and whole numbers. (a)
- Create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals. (b)
- Create and solve single-step practical problems involving division of decimals. (b)

5.5 **The student will**

a) estimate and determine the product and quotient of two numbers involving decimals* and

b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and create and solve single-step practical problems involving division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

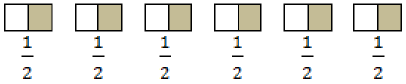
Understanding the Standard	Essential Knowledge and Skills
• Examples of appropriate decimal division problems for grade five students include, but are not limited to: – 2.38 ÷ 4; 6 ÷ 0.2; 1.78 ÷ 0.5; etc. – A scientist collected three water samples from local streams. Each sample was the same size, and she collected 1.35 liters of water in all. What was the volume of each water sample? – There are exactly 12 liters of sports drink available to the tennis team. If each tennis player will be served 0.5 liters, how many players can be served? – The relay team race is exactly 4.8 miles long. Each person on the team is expected to run 0.8 miles. How many team members will be needed to cover the total distance? • Division with decimals is performed the same way as division of whole numbers. The only difference is the placement of the decimal point in the quotient. • When solving division problems, numbers may need to be expressed as equivalent decimals by annexing zeros. This occurs when a zero must be added in the dividend as a place holder. • The quotient can be estimated, given a dividend expressed as a decimal through thousandths (and no adding of zeros to the dividend during the division process) and a single-digit divisor. • Estimation can be used to check the reasonableness of a quotient. • Division is the inverse of multiplication; therefore, multiplication and division are inverse operations. • Terms used in division are *dividend, divisor*, and *quotient.* $\textit{dividend} \div \textit{divisor} = \textit{quotient}$ $\begin{array}{r} \underline{\textit{quotient}} \\ \textit{divisor}\overline{)\textit{dividend}} \end{array}$ $\frac{\textit{dividend}}{\textit{divisor}} = \textit{quotient}$ • There are a variety of algorithms for division such as repeated multiplication and subtraction. Experience with these algorithms may enhance understanding of the traditional long division algorithm.	

5.6 **The student will**

a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and

b) solve single-step practical problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction, with models.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
- A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor. - When the numerator and denominator have no common factors other than one, then the fraction is in simplest form. - Fractions having like denominators have the same meaning as fractions having common denominators. - Addition and subtraction with fractions and mixed numbers can be modeled using a variety of concrete and pictorial representations. - Estimation keeps the focus on the meaning of the numbers and operations, encourages reflective thinking, and helps build informal number sense with fractions. Students can reason with benchmarks to get an estimate without using an algorithm. Estimation can be used to check the reasonableness of an answer. - A mixed number has two parts: a whole number and a fraction. The value of a mixed number is the sum of its two parts. - A unit fraction is a fraction in which the numerator is one. - Models for representing multiplication of fractions may include arrays, paper folding, repeated addition, fraction strips or rods, pattern blocks, or area models. - Students should begin exploring multiplication with fractions by solving problems that involve a whole number and a unit fraction. - When multiplying a whole number by a fraction such as $6 \times \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: six groups the size of $\frac{1}{2}$ of the whole.  $6 \times \frac{1}{2} = \frac{6}{2} = 3$	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** - Solve single-step and multistep practical problems involving addition and subtraction with fractions (proper or improper) having like and unlike denominators and/or mixed numbers. Denominators in the problems should be limited to 12 or less (e.g., $\frac{5}{8} + \frac{1}{4}$, $\frac{5}{6} - \frac{2}{3}$, $3\frac{3}{4} + 2\frac{5}{12}$) and answers should be expressed in simplest form. (a) - Solve single-step practical problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction (e.g., $6 \times \frac{1}{3}$, $\frac{1}{4} \times 8$, $9 \times \frac{2}{3}$), with models. The denominator will be a factor of the whole number and answers should be expressed in simplest form. (b) - Apply the inverse property of multiplication in models. (For example, use a visual fraction model to represent $\frac{4}{4}$ or 1 as the product of $4 \times \frac{1}{4}$). (b)

5.6 **The student will**

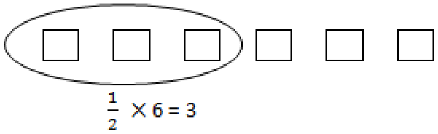
a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and

b) solve single-step practical problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction, with models.*

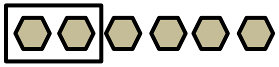
*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard

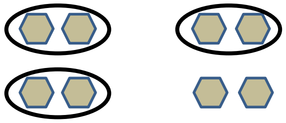
- When multiplying a fraction by a whole number such as $\frac{1}{2} \times 6$, we are trying to determine a part of the whole (e.g., one-half of six).



- The inverse property of multiplication states that every number has a multiplicative inverse and the product of multiplicative inverses is 1 (e.g., 5 and $\frac{1}{5}$ are multiplicative inverses because $5 \times \frac{1}{5} = 1$). The multiplicative inverse of a given number can be called the reciprocal of the number. Students at this level do not need to use the term for the properties of the operations.
- Multiplying a whole number by a unit fraction can be related to dividing the whole number by the denominator of the fraction. For example, $\frac{1}{3}$ of 6 is equivalent to 2. This understanding forms a foundation for learning how to multiply a whole number by a proper fraction.



- At this level, students will use models to solve problems that involve multiplication of a whole number, limited to 12 or less, and a proper fraction where the denominator is a factor of the whole number. For example, a model for $\frac{3}{4} \times 8$ or $8 \times \frac{3}{4}$ shows that the answer is three groups of $\frac{1}{4} \times 8$.



Essential Knowledge and Skills

5.6 The student will

a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and

b) solve single-step practical problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction, with models.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Examples of problems grade five students should be able to solve include, but are not limited to the following: – If nine children each bring $\frac{1}{3}$ cup of candy for the party, how many thirds will there be? What will be the total number of cups of candy? – If it takes $\frac{3}{4}$ cup of ice cream to fill an ice cream cone, how much ice cream will be needed to fill eight cones? • Resulting fractions should be expressed in simplest form. • Problems where the denominator is not a factor of the whole number (e.g., $\frac{1}{8} \times 6$ or $6 \times \frac{1}{8}$) will be a focus in grade six.	

5.7 The student will simplify whole number numerical expressions using the order of operations.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• An expression is a representation of a quantity. It is made up of numbers, variables, computational symbols, and grouping symbols. It does not have an equal symbol (e.g., 15 × 12). • Expressions containing more than one operation are simplified by using the order of operations. • The order of operations is a convention that defines the computation order to follow in simplifying an expression. It ensures that there is only one correct value. • The order of operations is as follows: – First, complete all operations within grouping symbols. If there are grouping symbols within other grouping symbols, do the innermost operation first. (Students in grade five are not expected to simplify expressions having parentheses within other grouping symbols.) – If there are multiple operations within the parentheses, apply the order of operations. – Second, evaluate all exponential expressions. (Students in grade five are not expected to simplify expressions with exponents.) – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use the order of operations to simplify whole number numerical expressions, limited to addition, subtraction, multiplication, and division. Expressions may contain parentheses. • Given a whole number numerical expression involving more than one operation, describe which operation is completed first, which is second, etc.

Students in grades three through five should be actively involved in measurement activities that require a dynamic interaction among students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, perimeter, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary) to measure length, weight/mass, liquid volume, area, perimeter, temperature, and time. Student understanding of measurement continues to be enhanced through experiences using appropriate tools such as rulers, balances, clocks, and thermometers.

The study of geometry helps students represent and make sense of the world. In grades three through five, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, draw representations of, and describe the relationships among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence; parallel, intersecting, and perpendicular lines; and classification of triangles.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of the parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

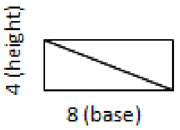
5.8 **The student will**

a) solve practical problems that involve perimeter, area, and volume in standard units of measure; and

b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.

Understanding the Standard

- A plane figure is any closed, two-dimensional shape.
- Perimeter is the path or distance around any plane figure. It is a measure of length.
- Area is the surface included within a plane figure. Area is measured by the number of square units needed to cover a surface or plane figure.
- Volume of a three-dimensional figure is a measure of capacity and is measured in cubic units.
- A polygon is a closed plane figure composed of at least three line segments that do not cross.
- To determine the perimeter of any polygon, add the lengths of the sides.
- Students should label the perimeter, area, and volume with the appropriate unit of linear, square, or cubic measure.
- A right triangle has one right angle.
- Students should use manipulatives to discover the formulas for the area of a right triangle and volume of a rectangular solid.
  - Area of a right triangle = $\frac{1}{2}$ base × height
  - Volume of a rectangular solid = length × width × height
- Students would benefit from opportunities that include the use of benchmark fractions (e.g., $\frac{1}{2}$. $\frac{1}{4}$) in determining perimeter.
- The area of a rectangle can be determined by multiplying the length of the base by the length of the height.
- The diagonal of the rectangle shown divides the rectangle in half creating two right triangles. The legs of the right triangles are congruent to the side lengths of the rectangle. The representation illustrates that the area of each right triangle is half the area of the rectangle. Exploring the decomposition of shapes helps students develop algorithms for determining area of various shapes (e.g., area of a triangle is ½ × base × height).



Essential Knowledge and Skills

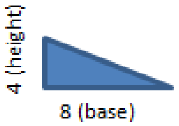
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Solve practical problems that involve perimeter, area, and volume in standard units of measure. (a)
- Determine the perimeter of a polygon, with or without diagrams, when
  - the lengths of all sides of a polygon that is not a rectangle or a square are given;
  - the length and width of a rectangle are given; or
  - the length of a side of a square is given. (a)
- Estimate and determine the area of a square and rectangle using whole number measurements given in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 24 square inches). (a)
- Develop a procedure for determining the area of a right triangle using only whole number measurements given in metric or U.S. Customary units, and record the solution with the appropriate unit of measure (e.g., 12 square inches). (a)
- Estimate and determine the area of a right triangle, with diagrams, when the base and the height are given. (a)
- Develop a procedure for determining volume using manipulatives (e.g., cubes). (a)
- Estimate and determine the volume of a rectangular prism with diagrams, when the length, width, and height are given, using whole number measurements. Record the solution with the appropriate unit of measure (e.g., 12 cubic inches). (a)

5.8 The student will

a) solve practical problems that involve perimeter, area, and volume in standard units of measure; and

b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.

Understanding the Standard	Essential Knowledge and Skills
• The distance from the top of the right triangle to its base is called the height of the triangle. • Two congruent right triangles can always be arranged to form a square or a rectangle.  • To develop the formula for determining the volume of a rectangular prism, volume = length × width × height, students will benefit from experiences filling rectangular prisms (e.g., shoe boxes, cereal boxes) with cubes by first covering the bottom of the box and then building up the layers to fill the entire box.	• Describe practical situations where perimeter, area, and volume are appropriate measures to use, and justify orally or in writing. (b) • Identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation. (b)

5.9 The student will

a) given the equivalent measure of one unit, identify equivalent measurements within the metric system; and

b) solve practical problems involving length, mass, and liquid volume using metric units.

Understanding the Standard	Essential Knowledge and Skills
• Length is the distance between two points along a line. • Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter ruler, meter stick, and tape measure. • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • Balances are appropriate measuring devices to measure mass in U.S. Customary units (ounces, pounds) and metric units (grams, kilograms). • Metric units to measure liquid volume (capacity) include milliliters and liters. • Practical experience measuring familiar objects helps students establish benchmarks and facilitates students' ability to use the appropriate units of measure to make estimates. • Students at this level will be given the equivalent measure of one unit when asked to determine equivalencies between units in the metric system. An example can be found below. – Students will be told 1 kilometer is equivalent to 1,000 meters and then will be asked to apply that relationship to determine: – the number of meters in 3.5 kilometers; – the number of kilometers equal to 2,100 meters; or – Seth ran 2.78 kilometers on Saturday. How many meters are equivalent to 2.78 kilometers?	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given the equivalent measure of one unit, identify equivalent measurements within the metric system for the following: – length (millimeters, centimeters, meters, and kilometers); – mass (grams and kilograms); and – liquid volume (milliliters and liters). (a) • Estimate and measure to solve practical problems that involve metric units: – length (millimeters, centimeters, meters, and kilometers); – mass (grams and kilograms); and – liquid volume (milliliters, and liters). (b)

5.10 The student will identify and describe the diameter, radius, chord, and circumference of a circle.

Understanding the Standard	Essential Knowledge and Skills
• A circle is a set of points in a plane that are the same distance from a point called the *center*. • A chord is a line segment connecting any two points on a circle. A chord may or may not go through the center of a circle. The diameter is the longest chord of a circle. • A diameter is a chord that goes through the center of a circle. The length of the diameter of a circle is twice the length of the radius. • A radius is a line segment joining the center of a circle to any point on the circle. Two radii end-to-end form a diameter of a circle. • Circumference is the distance around or "perimeter" of a circle. An approximation for circumference is about three times the diameter of a circle. An approximation for circumference is about six times the radius of a circle.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and describe the diameter, radius, chord, and circumference of a circle. • Investigate and describe the relationship between – diameter and radius; – diameter and chord; – radius and circumference; and – diameter and circumference.

5.11 The student will solve practical problems related to elapsed time in hours and minutes within a 24-hour period.

Understanding the Standard	Essential Knowledge and Skills
• Elapsed time is the amount of time that has passed between two given times. • Elapsed time can be found by counting on from the beginning time or counting back from the ending time.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve practical problems related to elapsed time in hours and minutes within a 24-hour period: – when given the beginning time and the ending time, determine the time that has elapsed; – when given the beginning time and amount of elapsed time in hours and minutes, determine the ending time; or – when given the ending time and the elapsed time in hours and minutes, determine the beginning time.

5.12 The student will classify and measure right, acute, obtuse, and straight angles.

Understanding the Standard	Essential Knowledge and Skills
- Angles can be classified as right, acute, obtuse, or straight according to their measures. - Angles are measured in degrees. A degree is $\frac{1}{360}$ of a complete rotation of a full circle. There are 360 degrees in a circle. - To measure the number of degrees in an angle, use a protractor or an angle ruler. - A right angle measures exactly 90 degrees. - An acute angle measures greater than zero degrees but less than 90 degrees. - An obtuse angle measures greater than 90 degrees but less than 180 degrees. - A straight angle measures exactly 180 degrees. - Before measuring an angle, students should first compare it to a right angle to determine whether the measure of the angle is less than or greater than 90 degrees. - Students should recognize angle measure as additive. When an angle is decomposed into nonoverlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. - Students should understand how to work with a protractor or angle ruler as well as available computer software to measure and draw angles and triangles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** - Classify angles as right, acute, obtuse, or straight. - Identify the appropriate tools (e.g., protractor and straightedge or angle ruler as well as available software) used to measure and draw angles. - Measure right, acute, obtuse, and straight angles, using appropriate tools, and identify their measures in degrees. - Solve addition and subtraction problems to determine unknown angle measures on a diagram in practical problems.

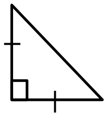
5.13 The student will

a) classify triangles as right, acute, or obtuse and equilateral, scalene, or isosceles; and

b) investigate the sum of the interior angles in a triangle and determine an unknown angle measure.

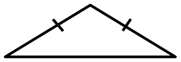
Understanding the Standard

- Angles can be classified as right, acute, obtuse, or straight according to their measures.
- A triangle can be classified as right, acute, or obtuse according to the measure of its largest angle.
- Triangles may also be classified according to the measure of their sides, e.g., scalene (no sides congruent), isosceles (at least two sides congruent) and equilateral (all sides congruent).
- An equilateral triangle (with three congruent sides) is a special case of an isosceles triangle (which has at least two congruent sides).
- Triangles can be classified by the measure of their largest angle and by the measure of their sides (i.e., an isosceles right triangle).



Isosceles Right Triangle

- Congruent sides are denoted with the same number of hatch (or hash) marks on each congruent side.
- A right angle measures exactly 90 degrees.
- An acute angle measures greater than zero degrees but less than 90 degrees.
- An obtuse angle measures greater than 90 degrees but less than 180 degrees.
- A straight angle measures exactly 180 degrees.
- A right triangle has one right angle.
- An obtuse triangle has one obtuse angle.
- An acute triangle has three acute angles.
- A scalene triangle has no congruent sides.
- An isosceles triangle has at least two congruent sides.



- An equilateral triangle has three congruent sides. All angles of an equilateral triangle are congruent and measure 60 degrees.

Essential Knowledge and Skills

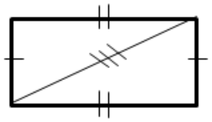
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Classify triangles as right, acute, or obtuse. (a)
- Classify triangles as equilateral, scalene, or isosceles. (a)
- Compare and contrast the properties of triangles. (a)
- Identify congruent sides and right angles using geometric markings to denote properties of triangles. (a)
- Use models to prove that the sum of the interior angles of a triangle is 180 degrees, and use that relationship to determine an unknown angle measure in a triangle. (b)

5.14 The student will

a) recognize and apply transformations, such as translation, reflection, and rotation; and

b) investigate and describe the results of combining and subdividing polygons.

Understanding the Standard	Essential Knowledge and Skills
• A transformation of a figure (preimage) changes the size, shape, or position of the figure to a new figure (image). Transformations can be explored using mirrors, paper folding, and tracing. • Congruent figures have the same size and shape. • A translation is a transformation in which an image is formed by moving every point on the preimage the same distance in the same direction. • A reflection is a transformation in which an image is formed by reflecting the preimage over a line called the *line of reflection*. All corresponding points in the image and preimage are equidistant from the line of reflection. • A rotation is a transformation in which an image is formed by rotating the preimage about a point called the center of rotation. The center of rotation may or may not be on the preimage. • The resulting figure of a translation, reflection, or rotation is congruent to the original figure. • The orientation of figures does not affect congruency or noncongruency. • A polygon is a closed plane figure composed of at least three line segments that do not cross. • Two or more polygons can be combined to form a new polygon. Students should be able to identify the figures that have been combined. • A polygon that can be divided into more than one basic figure is said to be a composite figure (or shape). Students should understand how to divide a polygon into familiar figures using concrete materials (e.g., pattern blocks, tangrams, geoboards, grid paper, paper (folding), etc.).  This diagonal of the rectangle above subdivides the rectangle in half and creates two right triangles. The figure can also be formed by combining two right triangles that are congruent. The resulting figure shows that the legs of the right triangles are congruent to the sides of the rectangle. The	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to** • Apply transformations to polygons in order to determine congruence. (a) • Recognize that translations, reflections, and rotations preserve congruency. (a) • Identify the image of a polygon resulting from a single transformation (translation, reflection, or rotation). (a) • Investigate and describe the results of combining and subdividing polygons. (b) • Compare and contrast the characteristics of a given polygon that has been subdivided with the characteristics of the resulting parts. (b)

5.14 The student will

a) recognize and apply transformations, such as translation, reflection, and rotation; and

b) investigate and describe the results of combining and subdividing polygons.

Understanding the Standard	Essential Knowledge and Skills
representation illustrates that the area of each right triangle is half the area of the rectangle. Exploring decomposition of shapes helps students develop algorithms for determining area of various shapes (e.g., area of a triangle is ½ × base × height). • Congruent sides are denoted with the same number of hatch (or hash) marks on each congruent side. For example, a side on a polygon with two hatch marks is congruent to the side with two hatch marks on a congruent polygon or within the same polygon.	

Students entering grades three through five have begun to explore the concept of the measurement of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students have had initial experiences in predicting whether a game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction in grades three through five is to deepen their understanding of the concepts of probability by:

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on:

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same? Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- communicating to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.

5.15 The student will determine the probability of an outcome by constructing a sample space or using the Fundamental (Basic) Counting Principle.

Understanding the Standard	Essential Knowledge and Skills
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives, tables, tree diagrams, and lists. • Probability is the measure of likelihood that an event will occur. An event is a collection of outcomes from an investigation or experiment. • The probability of an event can be expressed as a fraction, where the numerator represents the number of favorable outcomes and the denominator represents the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event is equal to: $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes.}}$ • Probability is quantified as a number between zero and one. An event is “impossible” if it has a probability of zero (e.g., the probability that the month of April will have 31 days). An event is “certain” if it has a probability of one (e.g., the probability that if today is Thursday then tomorrow will be Friday). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include practical examples. • A sample space represents all possible outcomes of an experiment. The sample space may be organized in a list, chart, or tree diagram. • Tree diagrams can be used to illustrate all possible outcomes in a sample space. For example, how many different outfit combinations can you make from two shirts (red and blue) and three pants (black, white, khaki)? The sample space displayed in a tree diagram would show the outfit combinations: red-black; red-white; red-khaki; blue-black; blue-white; blue-khaki. Exploring the use of tree diagrams for modeling combinations helps students develop the Fundamental Counting Principle. For this problem, applying the Fundamental Counting Principle shows there are 2 × 3 = 6 outcomes.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct a sample space, using a tree diagram to identify all possible outcomes. • Construct a sample space, using a list or chart to represent all possible outcomes. • Determine the probability of an outcome by constructing a sample space. The sample space will have a total of 24 or fewer equally likely possible outcomes. • Determine the number of possible outcomes by using the Fundamental (Basic) Counting Principle.

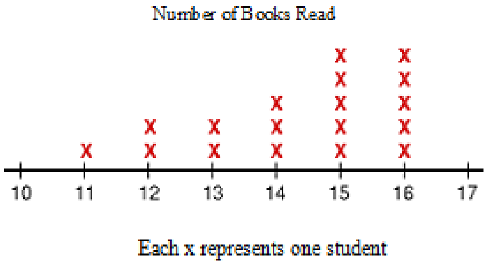
5.15 The student will determine the probability of an outcome by constructing a sample space or using the Fundamental (Basic) Counting Principle.

Understanding the Standard	Essential Knowledge and Skills
• The Fundamental (Basic) Counting Principle is a computational procedure to determine the total number of possible outcomes when there are multiple choices or several events. It is the product of the number of outcomes for each choice or event that can be chosen individually. For example, the possible final outcomes or outfits of four shirts (green, yellow, blue, red), two shorts (tan or black), and three shoes (sneakers, sandals, flip flops) is 4 × 2 × 3 = 24 outfits.	

5.16 **The student, given a practical problem, will**
a) represent data in line plots and stem-and-leaf plots;
b) interpret data represented in line plots and stem-and-leaf plots; and
c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.

Understanding the Standard

- The emphasis in all work with statistics should be on the analysis of the data and the communication of the analysis, rather than on a single correct answer. Data analysis should include opportunities to describe the data, recognize patterns or trends, and make predictions.
- Statistical investigations should be active, with students formulating questions about something in their environment and determining quantitative ways to answer the questions.
- Investigations that support collecting data can be brief class surveys or more extended projects taking many days.
- Through experiences displaying data in a variety of graphical representations, students learn to select an appropriate representation (i.e., a representation that is more helpful in analyzing and interpreting the data to answer questions and make predictions).
- There are two types of data: categorical and numerical. Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. While students need to be aware of the differences, they do not have to know the terms for each type of data.
- A line plot shows the frequency of data on a number line. Line plots are used to show the spread of the data and quickly identify the range and mode.



Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect data, using observations (e.g., weather), measurement (e.g., shoe sizes), surveys (e.g., hours watching television), or experiments (e.g., plant growth). (a)
- Organize the data into a chart or table. (a)
- Represent data in a line plot. Line plots will have no more than 30 data points. (a)
- Represent data in a stem-and-leaf plot where the stem is listed in ascending order and the leaves are in ascending order, with or without commas between leaves. Stem-and-leaf plots will be limited to no more than 30 data points. (a)
- Title the given graph or identify an appropriate title. (a)
- Interpret data by making observations from line plots and stem-and-leaf plots, describing the characteristics of the data and describing the data as a whole. One set of data will be represented on a graph. (b)
- Interpret data by making inferences from line plots and stem-and-leaf plots. (b)
- Compare data represented in a line plot with the same data represented in a stem-and-leaf plot. (c)

5.16 The student, given a practical problem, will
a) represent data in line plots and stem-and-leaf plots;
b) interpret data represented in line plots and stem-and-leaf plots; and
c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.

Understanding the Standard	Essential Knowledge and Skills

- A stem and leaf plot uses columns to display a summary of discrete numerical data while maintaining the individual data points. A stem-and-leaf plot displays data to show its shape and distribution.

Stem	Leaf
0	3,6,9
1	2,5,7,8
2	4,6
3	1,3,7,7,7
4	0,0,4,8
5	
6	1,2,2,3,8

3|5 = 35

  - The data are organized from least to greatest.
  - Each value is separated into a stem and a leaf (e.g., two-digit numbers are separated into stems (tens) and leaves (ones)).
  - The stems are listed vertically from least to greatest with a line to their right. The leaves are listed horizontally, also from least to greatest, and can be separated by spaces or commas. Every value is recorded, regardless of the number of repeats. No stem can be skipped. For example, in the stem and leaf plot above, there are no data for the stem 5; 5 should be listed showing no leaves.
  - A key is included to explain how to read the plot.
- Different situations call for different types of graphs. The way data are displayed is often dependent upon what someone is trying to communicate.
- Comparing different types of representations (e.g., charts graphs, line plots, etc.) provides students an opportunity to learn how different graphs can show different aspects of the same data. Following construction of representations, students benefit from discussions around what information each representation provides.

5.16 The student, given a practical problem, will

a) represent data in line plots and stem-and-leaf plots;

b) interpret data represented in line plots and stem-and-leaf plots; and

c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.

Understanding the Standard	Essential Knowledge and Skills
• Tables or charts organize the exact data and display numerical information. They do not show visual comparisons, which generally means it takes longer to understand or to examine trends. • Bar graphs can be used to compare data easily and see relationships. They provide a visual display comparing the numerical values of different categories. The scale of a bar graph may affect how one perceives the data. • Comparisons, predictions and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. • Sample questions that could be explored in comparing different representations such as a chart to a line plot and a stem-and-leaf plot could include: In which representation can you quickly identify the mode? The range? What predictions can you make?	

5.17 The student, given a practical context, will

a) describe mean, median, and mode as measures of center;

b) describe mean as fair share;

c) describe the range of a set of data as a measure of spread; and

d) determine the mean, median, mode, and range of a set of data.

Understanding the Standard	Essential Knowledge and Skills
• Statistics is the science of conducting studies to collect, organize, summarize, analyze, and draw conclusions from data. • Students need to learn more than how to identify the mean, median, mode, and range of a set of data. They need to build an understanding of what the measure tells them about the data, and see those values in the context of other characteristics of the data in order to best describe the results. • A measure of center is a value at the center or middle of a data set. Mean, median, and mode are measures of center. • The mean, median, and mode are three of the various ways that data can be analyzed. • The mean, median, and mode are referred to as types of averages. The term arithmetic average can be used when referring to the mean. • Mean represents a fair share concept of the data. Dividing the data constitutes a fair share. This idea of dividing as sharing equally should be demonstrated visually and with manipulatives to develop the foundation for the arithmetic process. The arithmetic way is to add all of the data points and then divide by the number of data points to determine the arithmetic average or mean. • The median is the middle value of a data set in ranked order. Given an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the arithmetic average of the two middle values. • The mode is the piece of data that occurs most frequently in the data set. There may be one, more than one, or no mode in a data set. Students should order the data from least to greatest so they can better determine the mode. • The range is the spread of a set of data. The range of a set of data is the difference between the greatest and least values in the data set. It is determined by subtracting the least number in the data set from the greatest number in the data set. An example is ordering test scores from least to greatest: 73, 77, 84, 87, 89, 91, 94. The greatest score in the data set is 94 and the least score is 73, so the least score is subtracted from the greatest score or 94 - 73 = 21. The range of these test scores is 21.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe and determine the mean of a group of numbers representing data from a given context as a measure of center. (a, d) • Describe and determine the median of a group of numbers representing data from a given context as a measure of center. (a, d) • Describe and determine the mode of a group of numbers representing data from a given context as a measure of center. (a, d) • Describe mean as fair share. (b) • Describe and determine the range of a group of numbers representing data from a given context as a measure of spread. (c, d)

Students entering grades three through five have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships within a pattern.

The focus of instruction is to help students develop a solid use of patterning as a problem-solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write “rules” for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

5.18 The student will identify, describe, create, express, and extend number patterns found in objects, pictures, numbers, and tables.

Understanding the Standard

- Mathematical relationships exist in patterns. There are an infinite number of patterns.
- Patterns and functions can be represented in many ways and described using words, tables, and symbols.
- Students need experiences exploring growing patterns using concrete materials and calculators. Calculators are valuable tools for generating and analyzing patterns. The emphasis is not on computation but on identifying and describing patterns.
- Patterns at this level may include: addition, subtraction, or multiplication of whole numbers; addition or subtraction of fractions (with denominators 12 or less); and decimals expressed in tenths or hundredths). Several sample numerical patterns are included below:
  - 1, 2, 4, 7, 11, 16, …;
  - 2, 4, 8, 16, 32, …;
  - 32, 30, 28, 26, 24…;
  - 0.15, 0.35, 0.55, 0.75…; and
  - $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}....$
- Students in grades three and four had experiences working with input/output tables to determine the rule or a missing value. Generalizing patterns to identify rules and applying rules builds the foundation for functional thinking. Sample input/output tables that require determination of the rule or missing terms can be found below:

Rule: ?	
Input	Output
4	8
5	?
6	12
?	20

Rule: ?	
Input	Output
8.9	9.4
6.6	7.1
?	3.5
0.5	1.0

- A numerical expression is a representation of a quantity. It is made up of numbers, variables, and/or computational symbols. It does not have an equal symbol (e.g., 15 × 12).

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify, create, describe, and extend patterns using concrete materials, number lines, tables, or pictures.
- Describe and express the relationship found in patterns, using words, tables, and symbols.
- Solve practical problems that involve identifying, describing, and extending single-operation input and output rules (limited to addition, subtraction and multiplication of whole numbers; addition and subtraction of fractions, with denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths).
- Identify the rule in a single-operation numerical pattern found in a list or table (limited to addition, subtraction and multiplication of whole numbers; addition and subtraction of fractions, with denominators of 12 or less; and addition and subtraction of decimals expressed in tenths or hundredths).

5.18 The student will identify, describe, create, express, and extend number patterns found in objects, pictures, numbers, and tables.

Understanding the Standard	Essential Knowledge and Skills
• A verbal expression involving one operation can be represented by a variable expression that describes the relationship. Numbers are used when they are known; variables are used when the numbers are unknown. The example in the table below defines the relationship between the input number and output number as *x* + 3. Students at this level are not expected to write a variable expression to describe patterns. They might describe the pattern below as + 3 or given any number, add three. • An algebraic expression is a variable or a combination of variables, numbers, and/or operation symbols and represents a mathematical relationship.	

x	y
6	9
7	10
11	14
15	18

5.19 The student will
a) investigate and describe the concept of variable;
b) write an equation to represent a given mathematical relationship, using a variable;
c) use an expression with a variable to represent a given verbal expression involving one operation; and
d) create a problem situation based on a given equation, using a single variable and one operation.

Understanding the Standard	Essential Knowledge and Skills
• A variable is a symbol that can stand for an unknown number (e.g., $a + 4 = 6$) or for a quantity that changes (e.g., the rule or generalization for the pattern for an input/output table such as $x + 2 = y$). • An algebraic expression, an expression with a variable, is like a phrase; a phrase does not have a verb, so an expression does not have an equal symbol (=). • A verbal expression describing a relationship involving one operation can be represented by an expression with a variable that mathematically describes the relationship. Numbers are used when quantities are known; variables are used when the quantities are unknown. For example, when *b* stands for the number of cookies in one full box, “the number of cookies in a full box and four extra” can be represented by $b + 4$; “three full boxes of cookies” by $3b$; “the number of cookies each person would receive if a full box of cookies were shared among four people” by $\frac{b}{4}$. • An equation is a statement that represents the relationship between two expressions of equal value (e.g., $12 \times 3 = 72 \div 2$). • A problem situation about two quantities that are equal can be expressed as an equation. • An equation may contain a variable and an equal symbol (=). For example, the sentence, “A full box of cookies and four extra equal 24 cookies.” can be written as $b + 4 = 24$, where *b* stands for the number of cookies in one full box. “Three full boxes of cookies contain a total of 60 cookies” can be written as $3b = 60$. • Another example of an equation is $b + 3 = 23$ and represents the answer to the word problem, “How many cookies are in a box if the box plus three more equals 23 cookies?” where *b* stands for the number of cookies in the box? • Teachers should consider varying the letters used (in addition to *x*) to represent variables. The symbol × is often used to represent multiplication and can be confused with the variable *x*. In addition to varying the use of letters as variables, this confusion can be minimized by using parentheses [e.g., $4(x) = 20$ or $4x = 20$] or a small dot raised off the line to represent multiplication [$4 \bullet x = 20$].	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the concept of a variable (presented as boxes, letters, or other symbols) as a representation of an unknown quantity. (a) • Write an equation with addition, subtraction, multiplication, or division, using a variable to represent an unknown quantity. (b) • Use an expression with a variable to represent a given verbal expression involving one operation (e.g., “5 more than a number” can be represented by $y + 5$). (c) • Create and write a word problem to match a given equation with a single variable and one operation. (d)

5.19 The student will

a) investigate and describe the concept of variable;
b) write an equation to represent a given mathematical relationship, using a variable;
c) use an expression with a variable to represent a given verbal expression involving one operation; and
d) create a problem situation based on a given equation, using a single variable and one operation.

Understanding the Standard	Essential Knowledge and Skills
• By using story problems and numerical sentences, students begin to explore forming equations and representing quantities using variables. • An equation containing a variable is neither true nor false until the variable is replaced with a number and the value of the expressions on both sides are compared.	